

- 2) the parallel translations g_a , where $a \in \mathbb{R}^2$ and $0 \leq |a| < 1$;
 3) the transformations $\lambda_0 : \Lambda \rightarrow \Lambda$, $\lambda_0(x) = x \operatorname{cth} b \operatorname{th}(\lambda b)$, where $\lambda \in \mathbb{R}$.

The following proposition is the main result of the paper.

Statement. *Let $\{A, B, C\}, \{A', B', C'\}$ be two triples of pairwise distinct points of the plane Λ . Then for sufficiently small $\varepsilon > 0$, applying the Maxima program, we find an element $g \in G$ such that $g(A) \in B(A', \varepsilon)$, $g(B) \in B(B', \varepsilon)$, $g(C) \in B(C', \varepsilon)$.*

Comment. 1. *We wrote the algorithm and the program in the built-in Maxima language in order to calculate the element $g \in G$ described in the statement.*

2. *This statement allows us to assume that the group G acts on the Lobachevsky plane in the Beltrami–Klein model 3-transitive.*

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О 3–ТРАНЗИТИВНОСТИ ГРУППЫ ПРЕОБРАЗОВАНИЙ ПЛОСКОСТИ ЛОБАЧЕВСКОГО В МОДЕЛИ БЕЛЬТРАМИ–КЛЕЙНА

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В статье в модели Бельтрами–Клейна плоскости Лобачевского рассматривается группа преобразований, которая является расширением группы движений. Устанавливается свойство этой группы, аналогичное свойству 3–транзитивности.

Ключевые слова: плоскость Лобачевского, метрика, группа преобразований.

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WORMHOLES

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In this paper we give an overview of wormhole solutions in the framework of the general theory of relativity.

Keywords: wormhole, theory of gravity, self-consistent solution.

By definition a wormhole is a bridge connecting two asymptotically flat regions. Usually such construction is a classical object and should satisfy to Einstein equations. The topology of the 4D wormholes is the topology of direct product of the Minkowski plane and a unit sphere. Static traversable wormholes could be threaded by "exotic matter" that violates certain energy conditions at least at the throat [1]. As an example of such a matter

one can consider the vacuum of quantized fields. This approach gives the possibility to consider the wormhole metric as the self-consistent solution of the semiclassical theory of gravity. In the realm of this theory the vacuum fluctuations of the quantized fields are the source of spacetime curvature¹

$$G_{\nu}^{\mu} = 8\pi \langle T_{\nu}^{\mu} \rangle_{ren}. \quad (1)$$

The main difficulty in the theory of semiclassical gravity is that the effects of the quantized gravitational field are ignored. The popular solution of this problem is to justify ignoring the gravitational contribution by working in the limit of a large number of fields, in which the gravitational contribution is negligible. Another problem is that the vacuum polarization effects are determined by the topological and geometrical properties of spacetime as a whole or by the choice of quantum state in which the expectation values are taken. It means that calculating of the functional dependence of $\langle T_{\nu}^{\mu} \rangle_{ren}$ on the metric tensor in an arbitrary spacetime presents formidable difficulty. Only in some spacetimes with high degrees of symmetry for the conformally invariant fields $\langle T_{\mu\nu} \rangle_{ren}$ can be computed and equations (1) can be solved exactly [3]. Let us stress that the single parameter of length dimensionality in such a problem is the Planck length l_{pl} . This implies that the characteristic scale l of the spacetime curvature (which correspond to the solution of equations (1)) can differ from l_{pl} only if there is a large dimensionless parameter. As an example of such a parameter one can consider a number of fields the polarization of which is a source of spacetime curvature². In some cases $\langle T_{\mu\nu} \rangle_{ren}$ is determined by the local properties of a spacetime and it is possible to calculate the functional dependence of the renormalized expression for the vacuum expectation value of the stress-energy tensor operator of the quantized fields on the metric tensor approximately. One of the most widely known examples of such a situation is the case of a very massive field. In this case $\langle T_{\nu}^{\mu} \rangle_{ren}$ can be expanded in terms of powers of the small parameter

$$\frac{1}{ml} \ll 1, \quad (2)$$

where m is the mass of the quantized field and l is the characteristic scale of the spacetime curvature [4, 5]. Using the first nonvanishing term of this expansion for minimally or conformally coupled scalar field Taylor, Hiscock and Anderson [6] have showed that the equations (1) have no wormhole solution for some class of static spherically symmetric spacetimes. Let us stress that in this case the existence of an additional parameter of the length dimensionality $1/m$ does not increase the characteristic scale of the spacetime curvature which is described by the solution of equations (1)³.

It is necessary to note that different quantum field theory constraints on the parameters of traversable wormholes are known [7]. As a rule these constraints were derived for a massless scalar field.

The purpose of this work is to examine whether the vacuum fluctuations of quantized fields in the one-loop approximation in Einstein's theory can create traversable

¹ Throughout we use units such that $c = \hbar = G = 1$.

² Here and below it is assumed, of course, that the characteristic scale of change of the background gravitational field is sufficiently greater than l_{pl} so that the very notion of a classical spacetime still has some meaning.

³ The characteristic scale of the components G_{ν}^{μ} on the left-hand side of equations (1) is $1/l^2$, on the right-hand side - $l_{pl}^2/(m^2 l^6)$.

Lorentzian wormholes. One of the possible ways to solve this problem is to use the analytical approximation for the expectation value of the stress-energy tensor operator of the quantized matter fields in curved spacetimes [5, 8]. This approach was realized in the works [9]. The problem of such an approach is the uncertainty of the applicability limits of such approximations. As it was noted by Khatsymovsky [10] in the spacetime that is a direct product of the Minkowski plane and a two-dimensional sphere of a fixed radius (the topologies of this spacetime and wormhole spacetime coincide) these approximations are not applicable. Another way to obtain the wormhole solution of equations (1) is to use the model of short-throat flat-space wormhole [11] (see also [12]). This model represents two identical copies of Minkowski spacetime with spherical regions excised from each copy and so that points of these regions are identical. One can consider this model as the first approximation of real situation if there is a small parameter L/r , where L is the length of the throat and r is the radius of the throat. At the present time only the full vacuum energy of quantized scalar field have been calculated in this spacetime. Nevertheless it gives a possibility to make some evaluations of the radius of a wormhole throat. A completely opposite model is the model of a long-throat wormhole. Local approximations for $\langle T_{\nu}^{\mu} \rangle$ in the throat of static spherically symmetric long-throat wormhole were obtained in [13] for massless fields of spin 1 and 1/2. The results of these works were obtained by the WKB method and the small parameter of these approximations is the ratio of the throat radius to the length of the throat. This result gives the possibility to reply to the question: can the throat of a long-throat wormhole be created by the vacuum fluctuations of quantum fields?

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КРОТОВЫЕ НОРЫ

А.А. Попов

В этой работе дается обзор решений, описывающих кротовые норы в рамках общей теории относительности.

Ключевые слова: кротовые норы, теория гравитации, самосогласованные решения.